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Transformations from cone responses to opponent color spaces

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Abstract

We have described a number of transformations by mapping the cone signal vector C, formed by LMS, to opponent space O, formed by achromatic signal O_A, red-green signal O_{rg}, and yellow-blue signal O_{yb}. Two new transformations, Γ_{op1} and Γ_{op2}, are proposed, based on the CIE 2006 spectral luminous efficiency function, $V_F(\lambda)$, and Hurvich's opponent spectral red-green and yellow-blue responses, $H_{rg}(\lambda)$ and $H_{yb}(\lambda)$, without and with constraints, respectively. Γ_{op1} is better than Γ_{op2} in terms of best fit of $H_{rg}(\lambda)$ and $H_{yb}(\lambda)$, and both transformations improved previously proposed ones. Finally, we discuss the *neutral conditions* in cone space using each of the new and some of the extant transformations, as well as choosing scaling factors so that the *neutral conditions* in cone space are satisfied.

KEYWORDS

cone fundamentals, opponent color space, opponent spectral responses

1 | INTRODUCTION

In 2006, the International Commission on Illumination (CIE) recommended 2 and 10° cone fundamental responses.[1-2] Figure 1 shows the CIE 2006 2° $\bar{l}(\lambda)$ (red), $\overline{m}(\lambda)$ (green), and $\bar{s}(\lambda)$ (blue) cone fundamental spectral responses after normalization (ie, when each of the responses has a unity peak). For a given stimulus with spectral radiance $\varphi(\lambda)$, cone response signals L, M, and S are computed from cone fundamental responses using the following equation:

$$\begin{cases} L = \int_a^b \varphi(\lambda)\bar{l}(\lambda)d\lambda \\ M = \int_a^b \varphi(\lambda)\overline{m}(\lambda)d\lambda \\ S = \int_a^b \varphi(\lambda)\bar{s}(\lambda)d\lambda \end{cases} \quad (1)$$

where a and b are the lower and upper limits, respectively, of the visible range. Achieving cone response signals L, M, and S for the spectral radiance $\varphi(\lambda)$ of a certain stimulus is the first stage for all two-step vision theories. The second stage is to transfer cone signals to an opponent color space, that is, an achromatic signal,

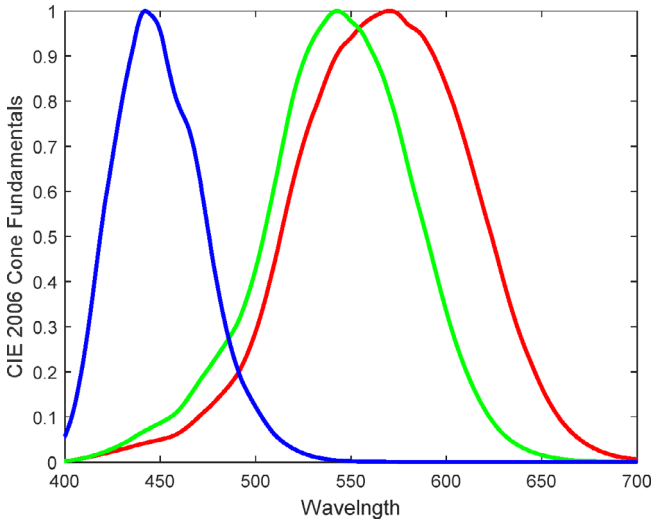

FIGURE 1 CIE 2006 2° cone fundamentals $\bar{l}(\lambda)$ (red), $\overline{m}(\lambda)$ (green), and $\bar{s}(\lambda)$ (blue), ranging from 400 nm to 700 nm

O_A, and two opponent chromatic signals, O_{rg} (red-green) and O_{yb} (yellow-blue). If we let the Γ transform be a three-by-three matrix, the second stage can be described as a matrix and vector multiplication:

$$\mathrm{O} = \Gamma \mathrm{C} \tag{2}$$

where vector C is formed by cone response signals, vector O is formed by signals in opponent color space, and the superscript T means transpose:

$$C = (L\ M\ S)^T \tag{3}$$

and

$$O = (O_A\ \ O_{rg}\ \ O_{yb})^T. \tag{4}$$

Different Γ transforms have been proposed in previous studies. For example, from figure 3 in the paper by Stockman and Brainard,[3] we have

$$\Gamma_{SB} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & 1 & -1 \end{pmatrix}. \tag{5}$$

From figure 1.4 in the book by Hunt and Pointer[4] we have

$$\Gamma_{HP} = \begin{pmatrix} 2 & 1 & 1/20 \\ 1 & -1 & 0 \\ 1 & 1 & -2 \end{pmatrix}. \tag{6}$$

From figure 1(8.3.4), p. 647, in the book by Wyszecki and Stiles,[5] we have

$$\Gamma_{WS} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix}. \tag{7}$$

In 2020, Wuerger et al[6] proposed

$$\Gamma_W = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -\dfrac{L_0}{M_0} & 0 \\ 1 & 1 & -\dfrac{L_0 + M_0}{S_0} \end{pmatrix}, \tag{8}$$

where L_0, M_0, and S_0 are the cone response signals for the neutral background under the given illuminant.

In 2021, from similarities in wavelength peaks in 16 studies of cone responses and 15 studies of opponent chromatic responses, Pridmore[7] proposed the following matrix for obtaining the two color opponent signals, O_{rg} and O_{yb}, from vector C in Equation (3):

$$\Gamma_P = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & -1 \end{pmatrix} \tag{9}$$

Recently, the CIE has set up its Technical Committee (TC) 1-98, "A roadmap towards basing CIE colorimetry on cone fundamentals," for studying the establishment of a new colorimetry based on cone response signals. As we have learned from current CIE colorimetry, developed from CIE color-matching functions, an opponent color space (eg, the CIELAB color space) is a key tool for the theoretical and practical applications of color science.[8] The currently unsolved problem is which transform should be used for transforming cone response signals to opponent color signals. In this article, we want to evaluate the above-mentioned transforms in terms of their fitting both the CIE spectral luminous efficiency function and the opponent chromatic spectral response signals reported by Hurvich.[9] Figure 2 shows Hurvich's opponent signals: $H_{rg}(\lambda)$ (solid curve) is the red-green spectral response and $H_{yb}(\lambda)$ (dotted curve) is the yellow-blue spectral response. More specifically, by using these

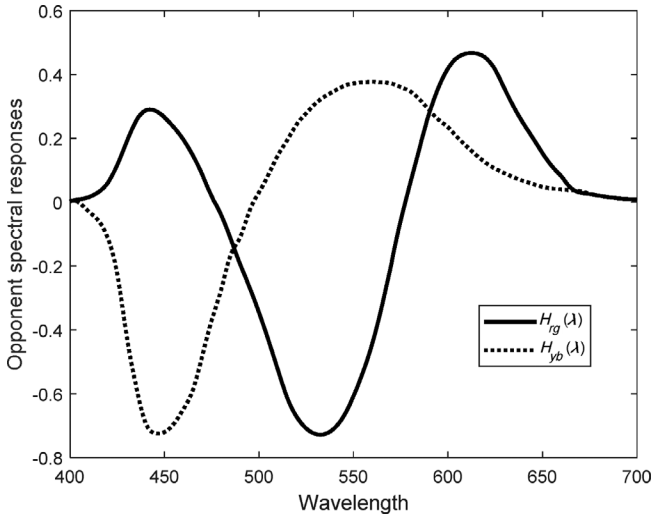

FIGURE 2 Opponent red-green $H_{rg}(\lambda)$ and yellow-blue $H_{yb}(\lambda)$ spectral responses redrawn from Hurvich[9]

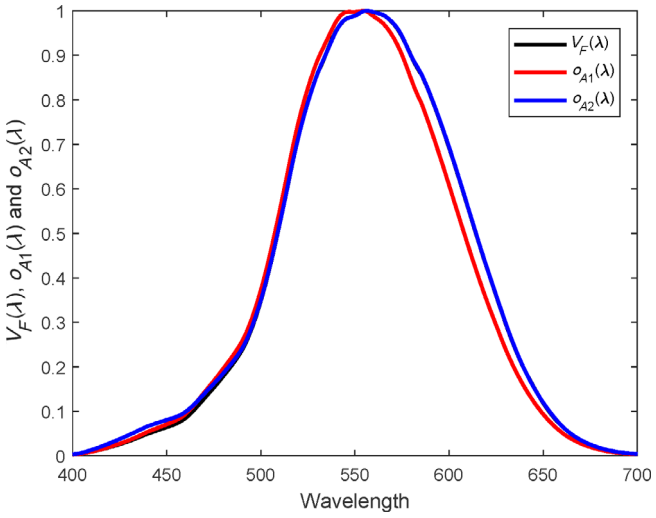

FIGURE 3 The CIE spectral luminous efficiency function $V_F(\lambda)$ (black), plus two normalized (unity peak) functions, $o_{A1}(\lambda)$ (red) and $o_{A2}(\lambda)$ (blue) (see main text), ranging from 400 nm to 700 nm

valuable experimental data sets as references, in the current paper we will develop two new Γ transforms (see Equation (2)), and discuss the *neutral conditions* in cone space for using each of the transforms defined by Equations (5)-(9) and the new transforms. We will also discuss how to choose scaling factors so that the *neutral conditions* in cone space are satisfied.

2 | NEW TRANSFORMS

Firstly, from Equation (2), if we let

$$C = c(\lambda) = \left(\bar{l}(\lambda) \;\; \overline{m}(\lambda) \;\; \bar{s}(\lambda)\right)^T, \tag{10}$$

the achromatic and opponent O signals are also dependent on the wavelength. Hence, we will let

$$O = o(\lambda) = \left(o_A(\lambda) \;\; o_{rg}(\lambda) \;\; o_{yb}(\lambda)\right)^T, \tag{11}$$

Since O_A is the achromatic signal in the opponent color space, we can expect $o_A(\lambda)$ to be close to the spectral luminous function. Several spectral luminous efficiency functions are available, including the Judd extension,[10] the CIE 1931 color-matching function $\bar{y}(\lambda)$, and the $V_F(\lambda)$, based on CIE 2006 2° cone fundamentals.[1-2] Since this article focuses on cone fundamentals, we have chosen the $V_F(\lambda)$ function as our reference.

Let $o_{A1}(\lambda) = o_A(\lambda)$, obtained using the transforms defined by Equations (5), (7), or (8), and $o_{A2}(\lambda) = o_A(\lambda)$, obtained using the transform defined by Equation (6). Figure 3 shows $V_F(\lambda)$ (black), as well as the normalized (unity peak) $o_{A1}(\lambda)$ (red) and $o_{A2}(\lambda)$ (blue) functions. As we can see, the normalized $o_{A1}(\lambda)$ (red) and $o_{A2}(\lambda)$ (blue) functions are quite close to $V_F(\lambda)$ (black). In particular, $V_F(\lambda)$ (black) and $o_{A2}(\lambda)$ (blue) nearly overlap for wavelengths greater than 475 nm. In fact, from CIE,[1-2] we have:

$$V_F(\lambda) = 0.68990272\bar{l}(\lambda) + 0.34832189\overline{m}(\lambda). \tag{12}$$

Hence, if the desired Γ transform (Equation (2)) has the general form of

$$\Gamma = \begin{pmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{pmatrix}, \tag{13}$$

then the best choice for the first row of Γ is:

$$\left(t_{11} \;\; t_{12} \;\; t_{13}\right) = \left(0.68990272 \;\; 0.34832189 \;\; 0\right). \tag{14}$$

Next, we will consider the two opponent red-green, O_{rg}, and yellow-blue, O_{yb}, signals. When we consider that $C = c(\lambda)$, defined by Equation (10), we can expect that $o_{rg}(\lambda)$ and $o_{yb}(\lambda)$ (see Equation (11)) will be close to the red-green $H_{rg}(\lambda)$ and yellow-blue $H_{yb}(\lambda)$ spectral responses (see Figure 2), respectively.

Let us consider the red-green channel first. From Equations (5)-(8) and (13), the first case (denominated RG-Case1) for the transform is

$$(t_{21} \quad t_{22} \quad t_{23}) = (1 \quad -x \quad 0). \tag{15}$$

When $x = 1$, the situation is that of Equations (5)-(7), and when $x = \frac{L_0}{M_0}$, the situation is that of Equation (8). We computed the best x so that $o_{rg}(\lambda)$ is closest to $H_{rg}(\lambda)$, and it was found that $x = 1.50273093$.

From Equation (9) (denominated RG-Case2) we have

$$(t_{21} \quad t_{22} \quad t_{23}) = (1 \quad -1 \quad 1). \tag{16}$$

In addition, we computed the best $(t_{21} \quad t_{22} \quad t_{23})$ so that $o_{rg}(\lambda)$ is closest to $H_{rg}(\lambda)$ (denominated RG-Case3), resulting in a least squares problem[11] with the following solution:

$$(t_{21} \quad t_{22} \quad t_{23}) = (1.19285019 \quad -1.74043682 \quad 0.35227109). \tag{17}$$

The fourth case (denominated RG-Case4) was to find the best $(t_{21} \quad t_{22} \quad t_{23})$ so that $o_{rg}(\lambda)$ is closest to $H_{rg}(\lambda)$ with constraint by the following Equation (18):

$$t_{21} + t_{22} + t_{23} = 0, \tag{18}$$

resulting in a constrained least squares problem[11] with the solution:

$$(t_{21} \quad t_{22} \quad t_{23}) = (1.20150523 \quad -1.67794634 \quad 0.47644111). \tag{19}$$

Note that the above constraint (Equation (18)) and the constraint considered later (Equation (25)) are for the Γ transform (see Equation (2)), mapping neutral in LMS cone space to neutral in opponent color space, with the neutral signals L_0, M_0, and S_0 satisfying $S_0 = L_0 = M_0$. This is widely used in the color and vision community[4-5] and will be further considered in the next section.

Figure 4 shows Hurvich's opponent red-green function $H_{rg}(\lambda)$ (black), and the red-green spectral responses achieved from RG-Case1 (red), RG-Case2 (blue), RG-Case3 (magenta), and RG-Case4 (yellow). It can be clearly seen that their order of closeness to the black curve is: magenta, yellow, red, and blue. Hence, RG-Case3 is the best, followed by RG-Case4, RG-Case1, and RG-Case2.

We can measure the closeness between $H_{rg}(\lambda)$ and the red-green spectral responses achieved by each of our four above cases by using different tools, such as the root mean square error (RMSE)[12] or the goodness of fit coefficient (GFC) used in previous literature.[13] Let q be a vector with n elements, and p an approximation to q, also

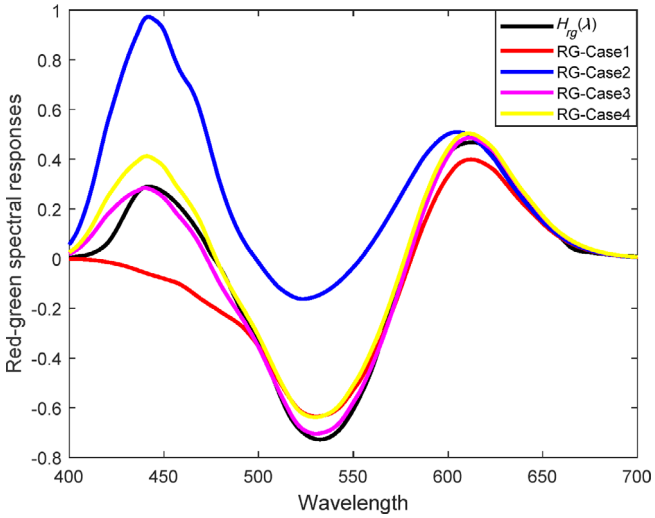

FIGURE 4 Hurvich's opponent red-green $H_{rg}(\lambda)$ function (black), together with the red-green spectral responses from RG-Case1, RG-Case2, RG-Case3, and RG-Case4, respectively (see main text)

with n elements. RMSE and GFC between q and p are defined by:

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(q_i - p_i)^2} \tag{20}$$

and

$$\text{GFC} = q^T p / (||q||_2 ||p||_2) \quad \text{with} \quad ||q||_2 = \sqrt{\sum_{i=1}^{n}(q_i)^2}, \quad ||p||_2 = \sqrt{\sum_{i=1}^{n}(p_i)^2}. \tag{21}$$

Thus, if $q = p$, RMSE = 0. The closer p and q are, the smaller RMSE is. For GFC, it is always equal or smaller than 1, and the closer to 1 the better the agreement between p and q. Table 1 lists the RMSE (column 2) and GFC (column 3) values between Hurvich's opponent red-green function $H_{rg}(\lambda)$ and the spectral responses achieved for each of our four cases. It can be seen from both measurements that the four cases, ordered from best to worst, are RG-Case3, RG-Case4, RG-Case1, and RG-Case2, which is in agreement with the results shown in Figure 4.

Note that from Equation (15) with x = 1.50273093 for RG-Case1, we cannot expect the transforms Γ_{SB}, Γ_{HP}, Γ_{WS}, and Γ_W (Equations (5)-(8)) to perform better than

TABLE 1 RMSE[12] and GFC[13] values between Hurvich's opponent red-green (RG) and yellow-blue (YB) functions and the four corresponding models (Case1-Case4) considered in the current article (see main text)

	RG		YB	
	RMSE	**GFC**	**RMSE**	**GFC**
Case1	0.139	0.920	0.800	0.754
Case2	0.391	0.475	0.369	0.907
Case3	**0.047**	**0.991**	**0.075**	**0.971**
Case4	0.085	0.970	0.127	0.914

Note: The numbers in bold indicate the best cases in the corresponding row in the first column under the measures shown in the row for RMSE and GFC in corresponding column.

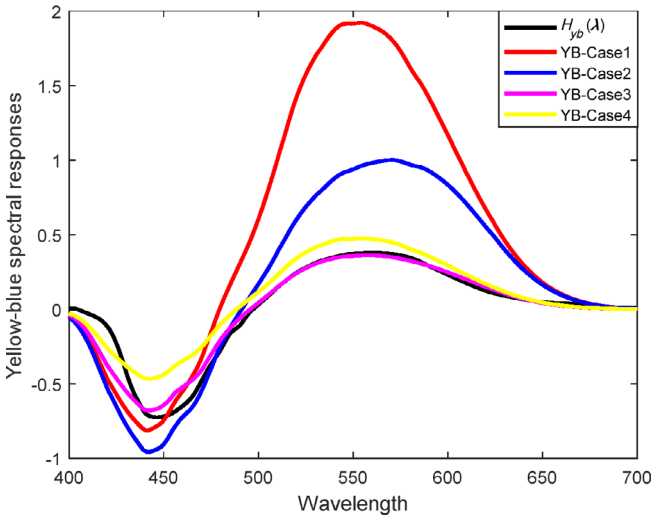

FIGURE 5 Hurvich's opponent yellow-blue $H_{yb}(\lambda)$ function (black), together with the yellow-blue spectral responses from YB-Case1, YB-Case2, YB-Case3, and YB-Case4, respectively (see main text)

RG-Case3 and RG-Case4, in terms of fitting Hurvich's opponent red-green function, $H_{rg}(\lambda)$. Similarly, RG-Case2, corresponding to the transform Γ_P (Equations (9) and (16)), is not expected to perform better than RG-Case3 and RG-Case4 in terms of fitting Hurvich's opponent red-green function $H_{rg}(\lambda)$.

We next considered the yellow-blue channel, and distinguished four cases as well. The first, denominated YB-Case1, is the following:

$$(t_{31} \quad t_{32} \quad t_{33}) = (1 \quad 1 \quad -y). \tag{22}$$

When $y = 1$, $y = 2$, and $y = -\frac{L_0+M_0}{S_0}$, we are presented with the situations indicated in previous Equations (5), (6), and (8), respectively. The best y value for $o_{yb}(\lambda)$, in order of closeness to $H_{yb}(\lambda)$, is $y = 0.92299502$.

The second case, denominated YB-Case2, is the following:

$$(t_{31} \quad t_{32} \quad t_{33}) = (1 \quad 0 \quad -1). \tag{23}$$

This case was given by Equation (7) and by Equation (9) in the recent paper[7] by Pridmore.

The third case, denominated YB-Case3, is to find the best $(t_{31} \quad t_{32} \quad t_{33})$ so that $o_{yb}(\lambda)$ is closest to $H_{yb}(\lambda)$. Again, we must solve a least squares problem[11] and it was found that

$$(t_{31} \quad t_{32} \quad t_{33}) = (0.24289310 \quad 0.13401208 \quad -0.69913752). \tag{24}$$

Finally, in the fourth case, denominated YB-Case4, we computed the best $(t_{31} \quad t_{32} \quad t_{33})$ so that $o_{yb}(\lambda)$ is closest to $H_{yb}(\lambda)$, with the constraint defined by the following Equation (25):

$$t_{31} + t_{32} + t_{33} = 0. \tag{25}$$

In this last case it was found that

$$(t_{31} \quad t_{32} \quad t_{33}) = (0.25717223 \quad 0.23710911 \quad -0.49428134). \tag{26}$$

As noted above, constraints in Equations (18) and (25) allow the Γ transform (see Equation (2)) to map the neutral in L, M and S space to the neutral in opponent space when the neutral stimulus signals L_0, M_0, and S_0 satisfy $S_0 = L_0 = M_0$.

Figure 5 shows Hurvich's opponent yellow-blue spectral function, $H_{yb}(\lambda)$ (black), together with the yellow-blue spectral responses found from YB-Case1 (red), YB-Case2 (blue), YB-Case3 (magenta), and YB-Case4 (yellow). It can be observed that their order of closeness to the black curve is: magenta, yellow, blue, and red. Hence, YB-Case3 is the best, followed by YB-Case4, YB-Case2, and YG-Case1.

Table 1 lists the RMSE (column 4) and GFC (column 5) values between Hurvich's opponent yellow-blue spectral function $H_{yb}(\lambda)$ and the yellow-blue spectral responses achieved for each of our four cases. It can be seen from both measurements that the best cases are YB-Case3, YB-Case4, YB-Case2, and YB-Case1, in this order, which is consistent with the results observed in Figure 5.

Note that from Equation (22) with $y = 0.92299502$ for YB-Case1 we cannot expect transforms Γ_{SB}, Γ_{HP}, and Γ_W

(Equations (5), (6), (8)) to perform better than for YB-Case3 and YB-Case4 in terms of fitting Hurvich's opponent yellow-blue function $H_{yb}(\lambda)$. In addition, for YB-Case2, corresponding to transforms Γ_{WS} and Γ_P, we cannot expect these to have a better performance than for YB-Case3 and YB-Case4 in terms of their fitting Hurvich's opponent yellow-blue function $H_{yb}(\lambda)$.

From all the above discussions, we can conclude that the best transform is based on Equation (14) for the achromatic signal, Equation (17) for the red-green signal, and Equation (24) for the yellow-blue signal. We have named this transform Γ_{op1}, given by:

$$\Gamma_{op1}=\begin{pmatrix} 0.68990272/k_L & 0.34832189/k_M & 0 \\ 1.19285019 & -1.74043682 & 0.35227109 \\ 0.24289310 & 0.13401208 & -0.69913752 \end{pmatrix}. \tag{27}$$

Similarly, we can point out that the second best transform, denominated Γ_{op2}, is the one achieved from Equations (14), (19), and (26):

$$\Gamma_{op2}=\begin{pmatrix} 0.68990272/k_L & 0.34832189/k_M & 0 \\ 1.20150523 & -1.67794634 & 0.47644111 \\ 0.25717223 & 0.23710911 & -0.49428134 \end{pmatrix}. \tag{28}$$

Here, k_L, k_M (and k_S) are scaling factors used to compute the cone response signals (see Equation (1)) that will be considered in the next section (see Equation (32)). Up to now, we have assumed that $k_L=1$ and $k_M=1$ have the best fit for the spectral luminous efficiency function $V_F(\lambda)$.

Note that the above analyses and proposed new transforms used the Hurvich's opponent response functions[9] as reference. Although different sets of opponent response functions are available in the literature (see Table 1 in reference 7), we used Hurvich's opponent response functions because they have been widely used in previous literature, including the recent article by Pridmore (figure 2 in reference 7).

3 | DISCUSSIONS AND CONCLUSIONS

In color science, an Γ transform is useful if $O=\Gamma C$ maps the neutral color vector $C_0=(L_0\ \ M_0\ \ S_0)^T$ to opponent space $O=(O_A\ \ O_{rg}\ \ O_{yb})^T$ with $O_{rg}=0$ and $O_{yb}=0$, which is referred to as a *neutral-to-neutral condition* for the Γ transform considered. It can be verified that the Γ_W transform given by Wuerger et al[6] (see Equation (8)) always satisfies this condition. However, for the other transforms described here we can deduce the *neutral condition* in cone space for the neutral color vector $C_0=(L_0\ \ M_0\ \ S_0)^T$ using this *neutral-to-neutral condition*.

For example, when using the transform Γ_{SB} defined by Equation (5), to obtain $O_{rg}=0$ and $O_{yb}=0$ we must have the *neutral condition* in cone space:

$$L_0=M_0 \text{ and } S_0=2L_0=2M_0. \tag{29}$$

Note that when using the transforms Γ_{HP}, Γ_{WS}, and Γ_{op2}, defined by Equations (6), (7), (28), respectively, the sum of the coefficients in the second and third rows of the matrices is zero, and hence, the *neutral-to-neutral condition* is fulfilled when the neutral color vector C_0 satisfies

$$S_0=L_0=M_0. \tag{30}$$

For the transform Γ_P, defined by Equation (9), the *neutral-to-neutral condition* is fulfilled if the neutral color vector C_0 satisfies

$$S_0=L_0, M_0=2L_0=2S_0. \tag{31}$$

To allow the neutral color stimulus to satisfy any of the neutral conditions defined by Equations (29)-(31), we must introduce the scaling factors k_L, k_M, and k_S into Equation (1) to compute the cone response signals *LMS*, so that

$$\begin{cases} L=k_L\int_a^b \varphi(\lambda)\bar{l}(\lambda)d\lambda \\ M=k_M\int_a^b \varphi(\lambda)\bar{m}(\lambda)d\lambda \\ S=k_S\int_a^b \varphi(\lambda)\bar{s}(\lambda)d\lambda \end{cases}. \tag{32}$$

Let $\varphi_0(\lambda)$ be the spectral radiance for a given neutral stimulus. Then, in order to satisfy the *neutral condition* Equation (29) in cone space, the scaling factors k_L, k_M, and k_S in Equation (32) are determined by the following Equation (33) for a given constant c (eg, $c=1$ or 100):

$$\begin{aligned} k_L\int_a^b \varphi_0(\lambda)\bar{l}(\lambda)d\lambda=k_M\int_a^b \varphi_0(\lambda)\bar{m}(\lambda)d\lambda=c \\ =0.5k_S\int_a^b \varphi_0(\lambda)\bar{s}(\lambda)d\lambda \end{aligned} \tag{33}$$

In order to satisfy the *neutral condition* Equation (30) in cone space, the scaling factors k_L, k_M, and k_S in Equation (32) are determined by the following Equation (34) for a given constant d (eg, $d=1$ or 100):

$$k_L\int_a^b \varphi_0(\lambda)\bar{l}(\lambda)d\lambda = k_M\int_a^b \varphi_0(\lambda)\overline{m}(\lambda)d\lambda = k_S\int_a^b \varphi_0(\lambda)\bar{s}(\lambda)d\lambda = d \tag{34}$$

However, in order to satisfy the *neutral condition* Equation (31) in cone space, the scaling factors k_L, k_M, and k_S in Equation (32) are determined by the following Equation (35) for a given constant f (eg, $f = 1$ or 100):

$$k_L\int_a^b \varphi_0(\lambda)\bar{l}(\lambda)d\lambda = k_S\int_a^b \varphi_0(\lambda)\bar{s}(\lambda)d\lambda = f = 0.5k_M\int_a^b \varphi_0(\lambda)\overline{m}(\lambda)d\lambda \tag{35}$$

Determining the scaling factors can also be done by using the newly derived transform Γ_{op1}. Let the second and third rows of the transform Γ_{op1} form a matrix A. Then we find the singular value decomposition[11] for A: $A = UDV^T$. Here, the superscript T is the transpose of a matrix, while U and V are 2-by-2 and 3-by-3 orthogonal matrices, respectively. The two-by-three matrix D has a special form, given by:

$$D = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \end{pmatrix} \tag{36}$$

where σ_1 and σ_2 are the singular (nonnegative) values of the matrix A.

Let v be the third column of the orthogonal matrix V. It can then be verified that $Av = (0 \;\; 0)^T$. In fact,

$$v = (0.73201193 \;\; 0.57550407 \;\; 0.36462803)^T. \tag{37}$$

Hence, the *neutral-to-neutral condition* using the transform Γ_{op1} is satisfied if the neutral color vector C_0 satisfies

$$C_0 = (L_0 \;\; M_0 \;\; S_0)^T = v. \tag{38}$$

In the preceding, we have discussed the *neutral-to-neutral conditions* for several different transforms. A transform is useful if the associated *neutral-to- neutral condition* is satisfied. We have also discussed the *neutral condition* in cone space for each of the transforms considered, and how to choose the scaling factors k_L, k_M, and k_S to satisfy these *neutral conditions*. However, different applications may require different *neutral conditions* in cone space, which may limit the choice of transforms. In color science, a widely accepted *neutral condition* is that the three channel response values L_0, M_0, S_0 are equal, resulting in the *neutral condition* in Equation (30), which ensures that transforms Γ_{HP} (see Equation (6)), Γ_{WS} (see Equation (7)), and Γ_{op2} (see Equation (28) satisfy the *neutral-to-neutral condition*. Among these three transforms, Γ_{op2} is the best in terms of fitting the spectral luminous efficiency function $V_F(\lambda)$ and Hurvich's opponent red-green and yellow-blue responses, $H_{rg}(\lambda)$ and $H_{yb}(\lambda)$.

When the three cone response values L_0, M_0, S_0 do not satisfy any of the *neutral conditions* given by Equations (29)-(31) and (38) in cone response space, for a particular application the transform Γ_W (see Equation (8)) can be used, as this transform does satisfy the *neutral-to-neutral condition* for any *neutral stimulus* in cone space. However, in this case, a better transform can be developed in terms of fitting of the spectral luminous efficiency function $V_F(\lambda)$ and Hurvich's opponent red-green and yellow-blue responses, $H_{rg}(\lambda)$ and $H_{yb}(\lambda)$. In fact, if we let the transform be Γ_{op3}, having 9 parameters (see Equation (13)), the first row of Γ_{op3} is the same as the first row of Γ_{op1} (or Γ_{op2}), and the second and third rows of Γ_{op3} should satisfy:

$$\begin{pmatrix} H_{rg}(\lambda) \\ H_{yb}(\lambda) \end{pmatrix} = \begin{pmatrix} t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{pmatrix} c(\lambda), \tag{39}$$

where $c(\lambda)$ is defined by Equation (10). Furthermore, the second and third rows of Γ_{op3} should satisfy the following constraint:

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{pmatrix} C_0, \tag{40}$$

where $C_0 = (L_0 \;\; M_0 \;\; S_0)^T$. Note that condition in Equation (39) must be satisfied for each wavelength λ, and hence we need to find a least squares solution for Equation (39). Condition in Equation (40) is the *neutral-to-neutral condition* for the transform Γ_{op3}, and so the final transform can be obtained by solving a constrained least squares problem.[11]

In conclusion, we have examined several transforms (see Equations (5)-(9)) mapping the cone signal vector C formed by LMS to opponent color space O (see Equations (2)-(4)). We have also derived two new transforms, designated as Γ_{op1} (Equation (27)) and Γ_{op2} (Equation (28)), based on the fit of the CIE spectral luminous efficiency function $V_F(\lambda)$ and Hurvich's opponent red-green and yellow-blue responses, $H_{rg}(\lambda)$ and $H_{yb}(\lambda)$, without and with constraints (see Equations (18) and

(25)), respectively. Γ_{op1} is better than Γ_{op2} in terms of best fit for $H_{rg}(\lambda)$ and $H_{yb}(\lambda)$. Both Γ_{op1} and Γ_{op2} are better than any of the transforms previously proposed (Equations (5)-(9)), in terms of fit of Hurvich's red-green and yellow-blue opponent spectral responses. Using the transform Γ_{op1}, the *neutral-to-neutral condition* requires that the scaling factors k_L, k_M, and k_S must be determined so that a *neutral stimulus* in cone space is obtained (Equations (37)-(38)). For the transform Γ_{op2}, the *neutral-to-neutral condition* requires that the scaling factors k_L, k_M, and k_S be determined so that the *neutral condition* in Equation (30) be satisfied. If the *neutral condition* in cone space is different from that defined by Equations (30) or (38), an Γ_{op3} transform can be obtained by solving the constrained least squares problem defined by Equations (39) and (40).

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DATA AVAILABILITY STATEMENT

The data used to support the findings of this study are available from the corresponding author upon request. The data are not publicly available due to privacy restrictions.

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