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RESEARCH ARTICLE

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Newton method for computing the adaptation coefficient in the CIE system of mesopic photometry

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Abstract
A study by Gao et al found that the Newton method may not converge for certain combinations of the photopic luminance and the ratio of scotopic and photopic luminance for computing the adaptation coefficient m, for the CIE MES2 system for mesopic photometry. Hence, they proposed to use the Bisection-Newton method. In this short note, we propose the Newton method with a new initial guess for computing m. Numerical simulation has shown the proposed method not only converges, but also converges faster than the Bisection-Newton method.

KEYWORDS
fixed-point iteration, MES2 system, mesopic vision, Newton method, photometry

1 | INTRODUCTION

The CIE spectral luminous efficiency functions $V(\lambda)$ for photopic and $V'(\lambda)$ for scotopic vision were standardized in 1924 and 1951, respectively. The wavelength dependent factors $V(\lambda)$ and $V'(\lambda)$ convert the radiant energy measures to luminance or photometric measures. Thus, the photopic luminance (denoted by L_p) and scotopic luminance (L_s) for a given spectral radiance $E(\lambda)$ (in W m^{-2} sr^{-1}) are defined by:

$$L_p = 683\int_{380}^{780} V(\lambda)E(\lambda)d\lambda, \quad L_s = 1699\int_{380}^{780} V'(\lambda)E(\lambda)d\lambda. \tag{1}$$

Note that the constant in Equation (1), 1699, was originally given as 1700.[1] In order to be consistent with the CIE MES2 system (see the constant, C, in Equations (5) and (6) below) the value 1700 is now replaced by 1699.

In 2010, CIE TC1-58 developed two variants (named as MES1 and MES2 respectively) for mesopic photometry, and finally based on visual perfromance CIE[1] recommended the MES2 system for computing the mesopic luminance L_{mes} (in cd m^{-2}). Firstly, the luminance efficient function $V_{mes}(\lambda)$ for mesopic vision with L_s and L_p satisfying[2]:

$$L_s > 0.005 \text{ cd } m^{-2} \quad \text{and} \quad L_p < 5.0 \text{ cd } m^{-2} \tag{2}$$

is defined as

$$M(m)V_{mes} = mV(\lambda) + (1-m)V'(\lambda), \tag{3}$$

where m is a coefficient of adaptation in the range $0 \le m \le 1$, $M(m)$ is a normalization constant such that $V_{mes}(\lambda)$ attains a maximum value of 1. Hence the mesopic luminance L_{mes} (in cd m^{-2}), for a given light source with a spectral radiance $E(\lambda)$ (in W m^{-2} sr^{-1}), is given by

$$L_{\mathrm{mes}} = \frac{683}{V_{\mathrm{mes}}(\lambda_0)} \int_{380}^{780} V_{\mathrm{mes}}(\lambda) E(\lambda) d\lambda, \tag{4}$$

where $\lambda_0 = 555$ nm. It can also be verified that

$$L_{\mathrm{mes}}(m) = \frac{mL_{\mathrm{p}} + (1-m)L_{\mathrm{s}}C}{m + (1-m)C} \tag{5}$$

with $C = V'(\lambda_0) = 683/1699$. However, to compute the mesopic luminance using Equation (4) or (5), the coefficient of adaptation m in Equation (3) must first be computed. If we let

$$F(m) = \frac{mL_{\mathrm{p}} + (1-m)L_{\mathrm{s}}C}{m + (1-m)C} - 10^{\frac{m-a}{b}}, \tag{6}$$

with

$$a = 0.7670, \quad b = 0.334, \tag{7}$$

then, the coefficient of adaptation m, is the solution of the equation $F(m) = 0$. Note that Gao et al[2] showed that the problem $F(m) = 0$ may have no solution, and may have more than one solution, which is certainly not desirable. Therefore, they suggested that a and b defined by Equation (7) should be redefined by:

$$a = 1 - \frac{\log_{10} 5}{3}, \quad b = \frac{1}{3}. \tag{8}$$

With the definition of a and b using Equation (8), Gao et al[2] proved the problem $F(m) = 0$ has a unique solution. So, from now on, with the CIE MES2 system, we assume the parameters a and b as defined using Equation (8).

Let

$$g(m) = a + b\log_{10}[L_{\mathrm{mes}}(m)], \tag{9}$$

where $L_{\mathrm{mes}}(m)$ is defined by Equation (5). Thus, $F(m) = 0$ is equivalent to $m = g(m)$. Therefore, the coefficient of adaptation m is a fixed point of the function $g(m)$. To compute the value of m, satisfying $m = g(m)$, the CIE[1] has recommended the fixed-point iteration method[3]

$$m_{n+1} = g(m_n), \quad \text{for} \quad n = 0, 1, \ldots \tag{10}$$

with $m_0 = 0.5$, until "convergence" or the inequality (11) defined below is achieved.

$$|m_{n+1} - m_n| \leq \varepsilon. \tag{11}$$

ε in the inequality Equation (11) is a predefined small tolerance. Therefore, when the inequality Equation (11) is achieved, the value of m_{n+1} is accepted as the solution of the equation $m = g(m)$.

It is clear that the function g, or the fixed-point iteration method, is dependent on both, L_{p} and the ratio $L_{\mathrm{s}}/L_{\mathrm{p}}$. In this article, the ratio $L_{\mathrm{s}}/L_{\mathrm{p}}$ will be denoted in an abbreviated form as S/P, that is,

$$S/P = L_{\mathrm{s}}/L_{\mathrm{p}}. \tag{12}$$

Recently, Shpak et al[4] and Gao et al[2] have reported that the convergence of the fixed-point iteration method depends on the ratio S/P, and for large values of S/P the method does not converge. Shpak et al[4] suspected that, to achieve convergence, the ratio S/P cannot be larger than 17. Let

$$C_1 = \frac{Cb}{\log 10} \approx 0.0582, \quad \text{and} \quad C_2 = \frac{1 + C_1}{C_1} \approx 18.1834. \tag{13}$$

Gao et al[5] showed that the fixed-point iteration method (Equation 10) is convergent for $S/P < C_2$, whenever a proper initial value m_0 is chosen. They also provided a better choice for the initial guess m_0. Let

$$\begin{aligned} g_0 &= g(0) = a + b\log_{10}(L_{\mathrm{s}}) \quad \text{and} \\ g_1 &= g(1) = a + b\log_{10}(L_{\mathrm{p}}). \end{aligned} \tag{14}$$

The new initial value m_0 is given by

$$m_0 = \begin{cases} 0.5(g_0 + g_1) & \text{if} \quad L_{\mathrm{s}} \leq 5 \\ 0.5(1 + g_1) & \text{if} \quad L_{\mathrm{s}} > 5 \end{cases}. \tag{15}$$

Up to now, one may ask how large can the ratio S/P be? Hung et al[6] considered the theoretical spectral radiance of a light source as a vector with 81 components sampled from 380 to 780 nm at 5 nm intervals, and then investigated the maximum luminous efficiency of radiation for a certain level of color rendering index and fixed correlated color temperature. Using a similar theoretical strategy, Gao et al[2] found the ratio S/P can be greater than 50. In fact, Figure 1 shows the S/P ratios for the monochromatic light using the Judd correction $V_{\mathrm{M}}(\lambda)$,[7] $\bar{y}(\lambda)$ from CIE 1931 color matching functions (CMFs), and $\bar{y}_{10}(\lambda)$ from CIE 1964 CMFs as the spectral luminous efficiency functions $V(\lambda)$. The S/P ratios depend on which set of $V(\lambda)$ is used. The maximum S/P ratio can be as large as 84.4 if $\bar{y}_{10}(\lambda)$ is used as $V(\lambda)$, up to 72.6 if $\bar{y}(\lambda)$ is used as $V(\lambda)$, and up to

24.5 if $V_M(\lambda)$ is used as $V(\lambda)$. Fotios and Yao[8] used $\bar{y}_{10}(\lambda)$ as $V(\lambda)$ for computing the S/P ratio.

From the above discussion, the S/P ratio can be larger than C_2 defined by Equation (13), hence the CIE recommended fixed-point iteration cannot be used when S/P is larger than C_2. Gao et al[2] considered the Newton method:

$$m_{n+1} = m_n - \frac{F(m_n)}{\frac{dF}{dm}(m_n)}, \quad \text{for} \quad n = 0, 1, \ldots \text{ with } \quad m_0 = 0.5 \tag{16}$$

for solving $F(m) = 0$ for the adaptation coefficient m. Here, $\frac{dF}{dm}(m_n)$ is the derivative of $F(m)$ evaluated at m_n. As is well-known, the convergence of the Newton method[3] depends on the initial guess m_0. If m_0 is in the convergence interval, the Newton method converges very fast. However, if m_0 is not a good choice, the Newton method may diverge. Gao et al[2] found that the Newton method does not converges for certain S/P ratios. Hence, they used the Bisection method[3] first to generate a sequence m_n, and in a later stage, the Newton method is used for fast convergence, resulting in the Bisection-Newton method[2] for computing the adaptation coefficient m for the CIE MES2 system.

2 | THE PROPOSED NEWTON METHOD

The Bisection-Newton method[2] was shown to be convergent. However, it is more complicated than the Newton method and it has a low convergence rate during the usage of the Bisection method. In this article, we use the initial guess m_0 defined by Equation (15) rather than using the Bisection method to get a better initial guess for the Newton method. The full procedure for the proposed Newton method is the following:

$$m_{n+1} = m_n - \frac{F(m_n)}{\frac{dF}{dm}(m_n)}, \quad \text{for} \quad n = 0, 1, \ldots \text{ with}$$
$$m_0 = \begin{cases} 0.5(g_0 + g_1) & \text{if} \quad L_s \le 5 \\ 0.5(1 + g_1) & \text{if} \quad L_s > 5 \end{cases}. \tag{17}$$

Note that $\frac{dF}{dm}(m_n)$, the derivative of the function $F(m)$ in Equation (17) is well defined and is given by Equation (18):

$$\frac{dF}{dm} = \frac{[L_p C(1 - L_s/L_p)]}{[m + (1-m)C]^2} - \frac{1}{b}\ln(10)10^{(m-a)/b} \tag{18}$$

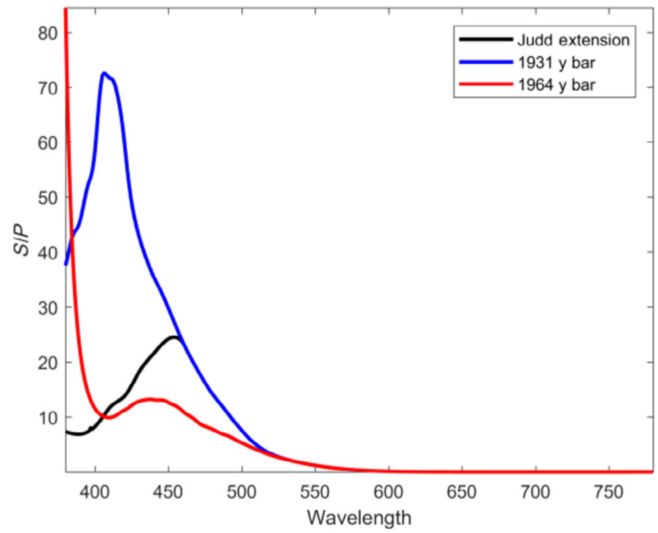

FIGURE 1 The S/P ratios for monochromatic lights between 380 and 780 nm when $V(\lambda)$ is chosen as the Judd extension, CIE 1931 $\bar{y}(\lambda)$, and CIE 1964 $\bar{y}_{10}(\lambda)$, respectively

It is expected that the above Newton method converges for all S/P ratios and converges faster than the Bisection-Newton method.

3 | PERFORMANCE OF THE PROPOSED NEWTON METHOD

In order to test our proposed method numerically, we have taken selected values for L_p from 0.1 to 4.9, namely 0.1, 0.3, 0.5, 0.7, ..., 4.9, that is a total number of 25 values for L_p. Similarly, we have taken selected values for the ratio S/P from 0.1 to 0.95, namely 0.1, 0.15, 0.2, 0.25, ..., 0.95, and from 1 to 85 at 1 unit steps, giving a total number of 61 values for the ratio S/P. Therefore, we have considered $1525 = 25 \times 61$ cases to test the performance of the proposed Newton method defined by Equation (17), together with the Newton method defined by Equation (16) considered by Gao et al,[2] and the Bisection-Newton method.[2] We have fixed tolerance $\varepsilon = 10^{-5}$ for the convergence, and have limited the number of iterations to 200, in order to avoid the program running for a long time.

Figures 2 and 3 show the contour plots of the Newton method defined Equation (16) and the proposed Newton method defined by Equation (17) in terms of the number of iterations used for each of the 1525 combinations of the photopic luminance L_p and ratio S/P. A decimal logarithmic scale was used in Figures 2 and 3 for S/P, to show more detail of the performance for smaller S/P values without discarding potential higher theoretical

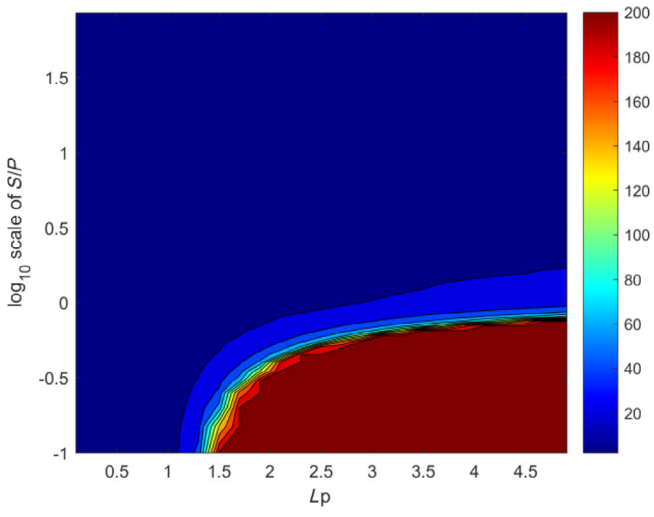

FIGURE 2 Contours plots with the number of iterations for the convergence of the Newton method defined Equation (16) considered by Gao et al,[2] as a function of S/P and L_p. Different colors represent different numbers of iterations needed, as shown on the vertical bar on the right

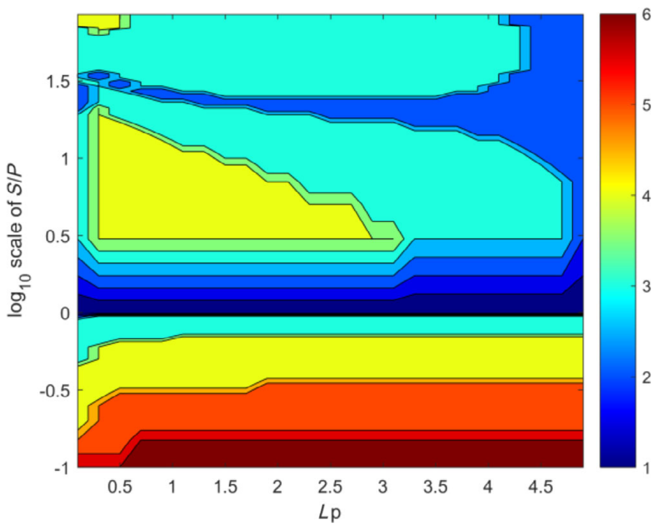

FIGURE 3 Contours plots showing the number of iterations for the convergence of the proposed Newton method defined by Equation (17), as a function of S/P and L_p. Different colors represent different numbers of iterations needed, as shown by the vertical bar on the right

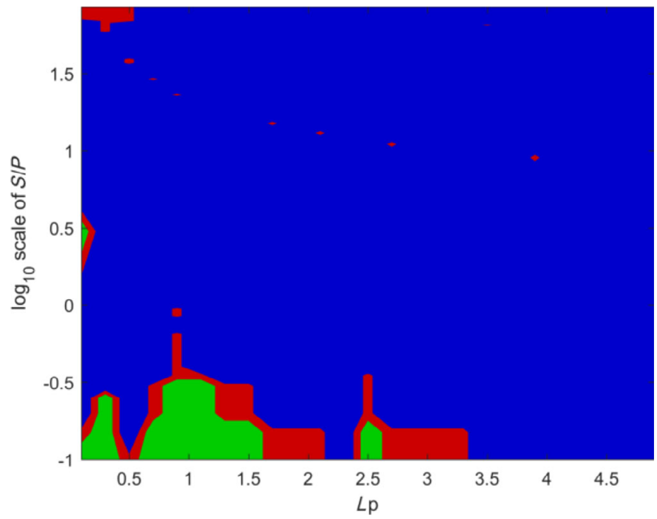

FIGURE 4 Contours plots for comparing the proposed Newton method (see Equation 17) with the Bisection-Newton method[2] in terms of number of iterations and the combinations of S/P and L_p in the blue area show the proposed Newton method is better than the Bisection-Newton method, in the red area show equal performance for both methods and in the green area show the Bisection-Newton method is better than the proposed Newton method

values. The numbers in the vertical color bars on the right of each of Figures 2 and 3 indicate the number of iterations needed for convergence. It can be seen, that for the combination of L_p and S/P in the lower-right red region of Figure 2, the Newton method defined Equation (16) did not converge after 200 iterations. And it can be seen from Figure 3 that the proposed Newton method defined Equation (17) converges for all the combinations of L_p and S/P and the maximum number for the convergence of the proposed Newton method is 6. Figure 4 shows the contour plot for comparing the proposed Newton method with the Bisection-Newton method[2] in terms of the difference between the numbers of iterations needed for each of the two methods. The combination of L_p and S/P in blue region shows the number of iterations needed for the proposed Newton method is less than the number of iterations needed for the Bisection-Newton method, and the green region shows the opposite. The red region shows the two methods need the same number of iterations for the convergence. It can be seen that the area of the blue region is much larger than the area of the red region, hence in general, the proposed Newton method converges faster than the Bisection-Newton method.

Furthermore, we also compared the proposed Newton method with the Bisection-Newton method in terms of CPU time using the 1525 samples. It was found that on average, the proposed Newton method took 61% of the CPU time needed for the Bisection-Newton method. Hence the proposed method is more efficient.

4 | CONCLUSION

In conclusion, we propose the Newton method with a new initial guess for m_0 defined by Equation (15). Numerical simulation has shown the proposed Newton method (see Equation 17) converges for all the combinations sampled from the theoretical region for the photopic luminance between 0.1 and 4.9, and for the S/P ratio between 0.1 and 85. Numerical simulation has also shown the proposed Newton method converges faster than the Bisection-Newton method and the proposed Newton method took about 61% of the CPU time needed for the Bisection-Newton method. Hence, it is recommended to use the proposed Newton method for computing the adaptation coefficient for the CIE MES2 system.[1]

DATA AVAILABILITY STATEMENT

The data used to support the findings of this study are available from the corresponding author upon request. The data are not publicly available due to privacy restrictions.

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REFERENCES

[1] CIE 191:2010. *Recommended System for Mesopic Photometry Based on Visual Performance*. Vienna, Austria: CIE; 2010.
[2] Gao C, Xu Y, Wang Z, et al. Improved computation of the adaptation coefficient in the CIE system of mesopic photometry. *Opt Express*. 2017;25(15):18365-18377.
[3] Burden RL, Faires JD. *Numerical Analysis*. 9th ed. Boston, USA: Brooks/Cole, Cengage Learning; 2011.
[4] Shpak M, Kärhä P, Ikonen E. Mathematical limitations of the CIE mesopic photometry system. *Light Res Technol*. 2017;49:111-121.
[5] Gao C, Zhang X, Xu Y, et al. Theoretical consideration on the convergence of the fixed-point iteration method in the CIE mesopic photometry system MES2. *Opt Express*. 2018;26(24):31351-31362.
[6] Hung PC, Tsao JY. Maximum white luminous efficacy of radiation versus color rendering index and color temperature: exact results and a useful analytic expression. *J Disp Technol*. 2013;9(6):405-412.
[7] CIE 86:1988. *2 Degree Spectral Luminous Efficient Functions for Photopic Vision*. Vienna, Austria: CIE; 2010.
[8] Fotios S, Yao Q. The association between correlated colour temperature and scotopic/photopic ratio. *Light Res Technol*. 2019;51(5):803-813.

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